

Optimum release angle in the shot put

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The aim of the study was to assess the accuracy of a method of calculating the optimum release angle in the shot put. Using the proposed method, the optimum release angle that produces the greatest flight distance is calculated by combining the equation for the range of a projectile in free flight with the relations between release speed, release height and release angle for the athlete. The method was evaluated using measurements of five college shot-putters who performed maximum-effort throws over a wide range of release angles. When the athletes threw with high release angles, the shot was released from a greater height above the ground and with a lower release speed. For all five athletes, the calculated optimum release angle was in good agreement with the athlete's preferred release angle. Each athlete had his own specific optimum release angle because of individual differences in the rate of decrease in release speed with increasing release angle. Simple models of shot-putting were developed to explain the relations between release speed, height and angle in terms of the anthropometric and strength characteristics of the athlete.

Keywords: athletics, projectile, release angle, shot put, sports biomechanics.

Introduction

In shot-putting, the athlete tries to project the shot as far as possible while remaining within the confines of the throwing circle. During the flight phase, the shot is essentially a projectile in free flight and so its trajectory can be accurately predicted given sufficient information about the release conditions (Hubbard, 1988; de Mestre, 1990). For a projectile that is projected with constant speed from above ground level, the optimum release angle that maximizes the horizontal range is always less than 45° . Lichtenberg and Wills (1978) calculated that the optimum release angle should be about 42° for a world-class throw of 22 m from a typical release height of 2.14 m.

It is well known that most world-class shot-putters use release angles that are lower than that predicted by Lichtenberg and Wills. Measured release angles have ranged from 26° to 45° , with an average value of about 37° (Dessureault, 1978; McCoy *et al.*, 1984; Susanka and Stepanek, 1988; Bartonietz and Borgström, 1995; Tsirakos *et al.*, 1995; Luhtanen *et al.*, 1997). Several investigators (Tricker and Tricker, 1967; Hay, 1973, 1993; Dyson, 1986; Hubbard, 1988; de Mestre, 1990; de Mestre *et al.*, 1998; Maheras, 1998) have suspected

that the disagreement arises because the release speed, height and angle are not independent, as was assumed in the calculation by Lichtenberg and Wills. Hay (1973) suggested that it may not be possible for a shot-putter to obtain the same release speed over a wide range of release angles. When shot-putters throw with a high release angle, they are opposed by a greater effect of gravity during the delivery phase, and so the release speed of the shot should be reduced. Also, the structure of the human body may favour the production of force (and hence release speed) in some directions more than others.

Red and Zogaib (1977) proposed a method of calculating the optimum projection angle in the javelin throw that accounted for the 'interaction' between the athlete and the implement during the delivery phase. In a study of three male javelin throwers, they observed a linear decrease in release speed with increasing release angle. The optimum release angle was calculated by combining the measured relation between release speed and release angle for the athlete with the equations describing the trajectory of the javelin in the flight phase. For the most experienced individual in the study, the calculated optimum release angle (37°) was in good agreement with measured release angles of world-class javelin throwers (35 – 38°).

The method of Red and Zogaib (1977) could also

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be used to calculate the optimum release angle in other throwing events, such as the shot put, discus throw and hammer throw. Yeadon (1998) developed a simple computer simulation model of shot-putting that assumed a linear decrease in release speed with increasing release angle. An optimum release angle of 37° was obtained by an appropriate selection of coefficients for the relation between release speed and release angle. To predict accurately the optimum release angle, the relation between the release height and release angle should also be included, but this is expected to have much less of an effect than the relation between release speed and release angle.

De Mestre *et al.* (1998) conducted an experimental study to establish whether the method of Red and Zogaib (1977) gives an accurate prediction of the optimum release angle in the shot put. A two-dimensional high-speed video analysis was used to measure the release speed, height and angle of three male college shot-putters who each threw the shot several times using a range of release angles (30 – 48°). Unfortunately, the results were not conclusive. For one of the athletes in the study, the predicted optimum release angle was within 1° of his preferred release angle, but for the other two athletes the discrepancies were 3° and 9° .

Maheras (1995) assessed optimum release angle in five male college shot-putters. The athletes in his study threw over a wider range of release angles (16 – 54°) than in the study by de Mestre *et al.* (1998) and for each athlete the relations between release speed, height and angle showed less inter-trial variation. Maheras found that release speed decreased linearly with increasing release angle and that release height increased linearly with increasing release angle. The athletes' preferred release angles (31 – 36°) were significantly lower than the optimum release angle (about 41°) calculated using the method of Lichtenberg and Wills (1978). Surprisingly, Maheras did not calculate an optimum release angle using the method of Red and Zogaib (1977).

The present study uses Maheras' (1995) measurements to assess the accuracy of Red and Zogaib's (1977) method of calculating the optimum release angle in the shot put. Maheras' measurements of release speed, height and angle for the five shot-putters are presented, together with simple models that relate the measurements to the anthropometric and strength characteristics of the athlete. The optimum release angle of each athlete is calculated by combining the equation for the flight distance of a shot in free flight with the relations between the release speed, height and angle for the athlete. The calculated optimum release angle of the athlete is then compared with his preferred release angle. In the next section, some background on shot-putting is presented and the effects of release

speed, height and angle on the range of a projectile in free flight are examined.

Shot-putting and projectile motion

Shot-putting requires great explosive strength, together with the ability to perform precisely timed movements in a confined space. The athlete's objective is to project the shot as far as possible, but competition regulations restrict the technique that may be used. The shot must be thrown from the shoulder using one hand and it must be held near to the chin throughout any preliminary movements (IAAF, 2000). Throughout the throwing motion, the athlete must remain within a circle of 2.135 m diameter that has a 10 cm high stop board placed at its front edge.

The two most widely used throwing techniques are the glide technique and the rotational technique (Fig. 1). These techniques differ in the preliminary movements that the athlete makes to move across the throwing circle, but the delivery phase is similar in both. During the delivery phase, the athlete exerts force on the shot with an explosive straightening of the legs, coupled with a raising and rotation of the trunk, followed by a rapid extension of the arm in the direction of the throw.

Shot put performance is quantified by the official distance, d_{official} , which is the distance from the inside of the circumference of the throwing circle to the nearest mark made by the fall of the shot. The official distance may be considered as the sum of the release distance, d_{release} , the flight distance, d_{flight} , and the landing distance, d_{landing} (Fig. 2). The release distance is the horizontal distance from the inside edge of the stop board to the centre of mass of the shot at the instant of release, the flight distance is the horizontal distance the centre of mass of the shot travels from the instant of release to the instant of landing, and the landing distance is the horizontal distance from the centre of mass of the shot at the instant of landing to the mark on the ground nearest to the throwing circle. In most throws, the flight distance is almost equal to the official distance. The release distance is usually small, but may be positive or negative, depending on how close the athlete's body is to the stop board and on the angle of the athlete's throwing arm to the horizontal. The landing distance is always a very small negative component of the official distance.

To achieve the greatest possible flight distance, the athlete must project the shot with the optimum combination of release speed, angle and height. Release speed is strongly correlated with throwing distance and is undoubtedly the most important factor. World-class shot-putters have release speeds of 12.5 – $14.5 \text{ m}\cdot\text{s}^{-1}$

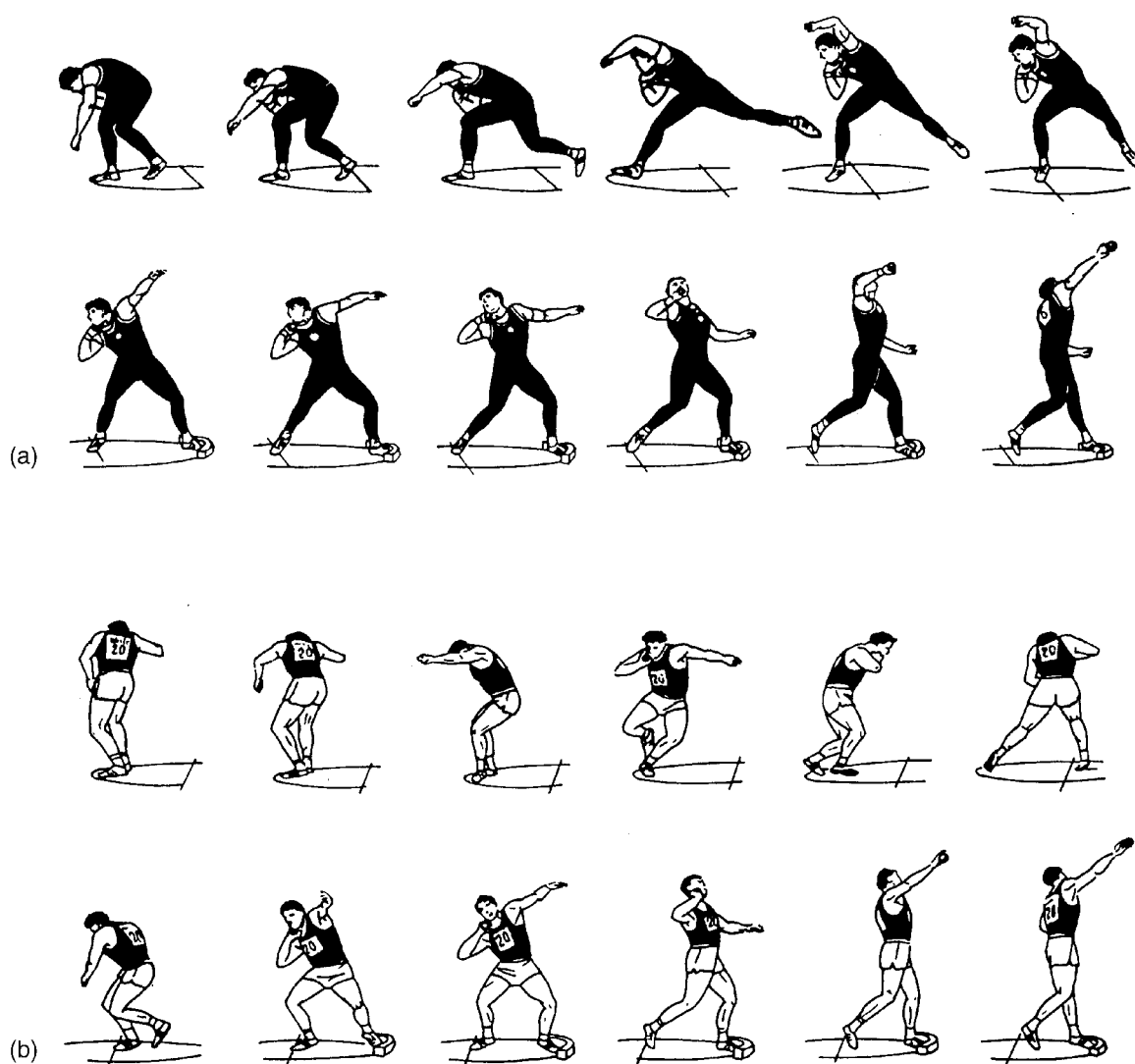


Fig. 1. Sequence of movements by a shot-putter using (a) the glide technique and (b) the rotational technique. Reproduced with permission from *Athletes in Action: The Official International Amateur Athletic Federation Book on Track and Field Techniques* (edited by H. Payne). London: Pelham Books.

and achieve distances of 19–23 m (Dessureault, 1978; McCoy *et al.*, 1984; Susanka and Stepanek, 1988; Tsirakos *et al.*, 1995; Luhtanen *et al.*, 1997).

The release angle is less important than the release speed, but substantial deviations from the optimum release angle may have an adverse effect on an athlete's performance. Studies of world-class shot-putters show release angles ranging from 26° to 45° , with an average value of about 37° (Dessureault, 1978; McCoy *et al.*, 1984; Susanka and Stepanek, 1988; Bartonietz and Borgström, 1995; Tsirakos *et al.*, 1995; Luhtanen *et al.*, 1997). Individual athletes may have marked differences in their average release angle, with values as low as 29° and as high as 38° .

The height of release also influences shot-put performance. The height of the shot above the ground at the instant of release is determined mainly by the height of the athlete and by the angle of the athlete's arm to the horizontal. World-class male athletes usually have a greater release height (average = 2.22 m) than female athletes (average = 2.07 m) because they are taller (Susanka and Stepanek, 1988).

The shot is a relatively heavy implement (7.26 kg for men, 4.00 kg for women) and, unlike the discus and javelin, it does not have any special flight-enhancing features. Aerodynamic drag and lift are small perturbations to the effects of gravity and so may be neglected (Lichtenberg and Wills, 1978; Hubbard, 1988; de

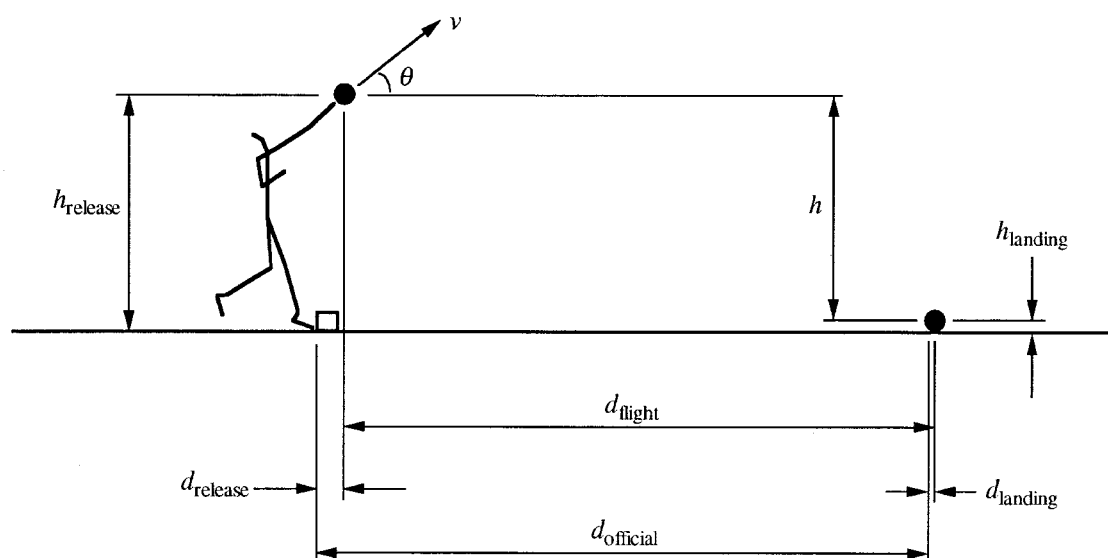


Fig. 2. Diagram of shot-putting showing contributions to the official distance.

Mestre, 1990). Similarly, the wind (Lichtenberg and Wills, 1978), spinning motion of the shot (de Mestre, 1990) and altitudinal variations in the acceleration due to gravity (Lichtenberg and Wills, 1978), all have a small influence on the trajectory of the shot. The shot, therefore, may be treated as a projectile in free flight.

Since the flight distance is very close to the official distance, the release angle that maximizes the flight distance is expected to be very close to the optimum release angle that maximizes the official distance. The flight distance of a shot in free flight is given by:

$$d_{\text{flight}} = \frac{v^2 \sin 2\theta}{2g} \left[1 + \left(1 + \frac{2gh}{v^2 \sin^2 \theta} \right)^{1/2} \right] \quad (1)$$

where v is the release speed, θ is the release angle and g is the acceleration due to gravity. The height difference between the release and the landing, h , is given by:

$$h = h_{\text{release}} - h_{\text{landing}} \quad (2)$$

where h_{release} is the release height and h_{landing} is the landing height (Fig. 2). For shot-putting on level ground, the landing height is equal to the radius of the shot (5.5–6.5 cm for men, 4.75–5.5 cm for women). When $h = 0$, equation (1) reduces to the familiar expression for the range of a projectile launched from ground level over a horizontal plane, $d_{\text{flight}} = (v^2 \sin 2\theta)/g$.

It is instructive to show in graphical form the application of equation (1) to the flight of a shot. Figure 3 shows the effect of changes in release angle on the flight distance of the shot. Curves are shown for a range of release speeds, from a value that produces a modest flight distance to a value slightly greater than that produced by a world-class shot-putter. The calculations

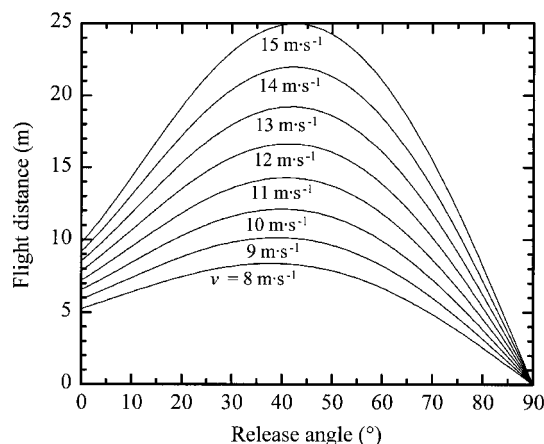


Fig. 3. The effect of release speed and release angle on the flight distance of a shot in free flight. The curves are with a constant release height, $h = 2.10$ m.

are made with $h = 2.10$ m, which is representative of the height difference for a world-class shot-putter. Some well-known features of projectile motion are illustrated by these curves. When the release speed is held constant, there is an optimum release angle that maximizes the flight distance. This optimum release angle is determined by a balance between the opposing effects of increased flight time and decreased horizontal speed as the release angle is raised. The optimum release angle is always less than 45° , but the optimum release angle approaches closer to 45° with increasing release speed. At the optimum release angle, the flight distance depends on the square of the release speed.

The radius of the shot is relatively small and so the height difference between the release and the landing

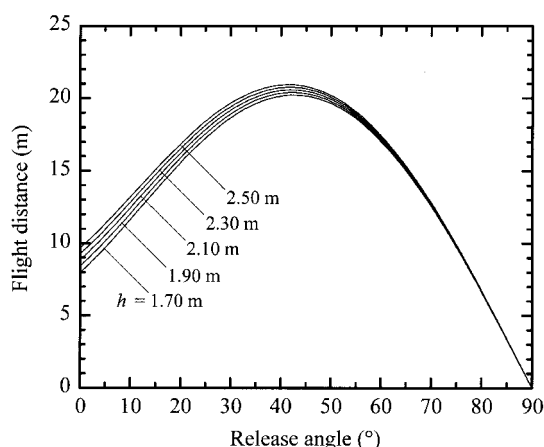


Fig. 4. The effect of release height and release angle on the flight distance of a shot in free flight. The curves are with a constant release speed, $v = 13.5 \text{ m} \cdot \text{s}^{-1}$.

is approximately equal to the release height. Figure 4 shows the effect of changes in release height on the flight distance of the shot. The calculations are made with a constant speed of $13.5 \text{ m} \cdot \text{s}^{-1}$, which is representative of that for a world-class shot-putter. Curves are shown for a range of release heights, from a value that would be expected for an athlete about 1.50 m tall ($h = 1.70 \text{ m}$) to that for an athlete about 2.20 m tall ($h = 2.50 \text{ m}$). A higher release height produces a greater flight distance, but at a slightly lower optimum release angle. In the shot put, release speed is considered more important than release height because changes in release speed have a greater effect on the flight distance than equal relative changes in release height (Hay, 1993).

The preceding discussion follows the approach taken by Lichtenberg and Wills (1978), in which the release speed, height and angle are treated as independent variables. The aim of the present study was to validate Red and Zogaib's (1977) proposal that the relations between the release speed, height and angle must be included in the calculation of the optimum release angle.

Methods

This study used the data reported by Maheras (1995) for the release speed, height and angle of five male college shot-putters. The athletes in Maheras' study were highly experienced shot-putters and so it was assumed that their preferred release angle would be very close to their optimum release angle. The accuracy of Red and Zogaib's (1977) method of calculating the optimum release angle was assessed by comparing the calculated optimum release angle with the athlete's preferred release angle.

The optimum release angle calculated using the method of Red and Zogaib (1977) was also compared with that calculated using the method of Lichtenberg and Wills (1978). Although Lichtenberg and Wills' method is known to produce an incorrect value, the calculation was included to highlight the difference between the two methods. In the method used by Lichtenberg and Wills (1978), the optimum release angle is calculated directly from the equation for the flight distance of a shot in free flight (equation 1). All that is required is knowledge of the athlete's release speed and height, which can be measured from high-speed video analysis of the release phase of a competition performance by the athlete. The measured release speed, v , and difference in height between release and landing, h , are substituted into equation (1) and the flight distance plotted as a function of the release angle. The athlete's optimum release angle is the point on the curve at which the flight distance is greatest. An alternative technique is to obtain a mathematical expression for the optimum release angle by finding the critical point with respect to release angle in equation (1). The athlete's optimum release angle is calculated by substituting the athlete's values of v and h into the equation for the critical point.

In the method proposed by Red and Zogaib (1977), the optimum release angle is calculated by combining the equation for the flight distance of a shot in free flight (equation 1) with the relations between the release speed, height and angle for the athlete. This method requires intervention by the investigator to obtain measurements of the athlete's release speed and height over a wide range of release angles, rather than just at the athlete's preferred release angle. The values for release speed and height are plotted as a function of the release angle, and curves are fitted to give algebraic expressions for release speed as a function of release angle, $v(\theta)$, and for the height difference as a function of release angle, $h(\theta)$. The expressions for $v(\theta)$ and $h(\theta)$ are then substituted into equation (1) and the flight distance plotted as a function of release angle. The optimum release angle is the point on the curve at which the flight distance is greatest. An alternative technique is to use the method of Lagrange multipliers to find the maximum value of the range of a projectile in free flight (equation 1), subject to the constraints of $v(\theta)$ and $h(\theta)$.

The mathematical expressions for $h(\theta)$ and $v(\theta)$ may be obtained by using the method of least-squares to fit a straight line or a polynomial to the plots of release height and release speed as a function of release angle. A more illuminating method is to derive the fitted mathematical expressions from physical models, with $h(\theta)$ and $v(\theta)$ being used to evaluate the models. In this study, the expression for $h(\theta)$ was derived from an anthropometric model of the athlete at the instant of release and the

expression for $v(\theta)$ was derived from a model of the forces acting on the shot during the delivery phase. These two models are described in the following sub-sections.

Model of the release height

At the instant of release, the athlete is assumed to be standing upright, as shown in Fig. 5. The athlete has the throwing shoulder raised so that the line of the shoulders is aligned with the throwing arm, and the angle of the throwing arm to the horizontal is the same as the release angle (Susanka and Stepanek, 1988). The expression for the height of the shot at the instant of release (i.e. the release height) is:

$$h_{\text{release}} = h_{\text{shoulder}} + l_{\text{arm}} \sin \theta \quad (3)$$

where h_{shoulder} is the height of the athlete's shoulders when standing upright and l_{arm} is the length of the athlete's outstretched throwing arm and shoulder. When shot-putting on level ground, the landing height is equal to the radius of the shot, and so the height difference between the release and the landing (equation 2) becomes:

$$h(\theta) = h_{\text{shoulder}} + l_{\text{arm}} \sin \theta - r_{\text{shot}} \quad (4)$$

where r_{shot} is the radius of the shot.

Models of the release speed

Figure 6 shows a model where the shot-putting action is reduced to just the delivery phase. The athlete applies to the shot a force, F , at an angle ϕ to the horizontal. The combined effect of the applied force and the weight of the shot, mg , is a resultant force, R , that produces acceleration of the shot along a straight line path, l , at an angle θ to the horizontal (Tricker and Tricker, 1967). This acceleration path is at the same angle to the horizontal as the release angle (Susanka and Stepanek, 1988).

The model does not consider the rotating or gliding movements of the athlete before the delivery phase because they have only a minor influence on the distance achieved. At the end of the preliminary movement, the speed of the shot is relatively low and the motion of the shot is directed horizontally, rather than in the same direction as the shot at release. It must be emphasized that the speed generated in the preliminary phase of the throw does *not* transfer completely to the delivery phase. To illustrate this point, Koltai (1974) and Zatsiorsky (1990) noted that an elite male shot-putter can throw the shot about 19.5 m without a preliminary movement (i.e. in a standing throw). The release speed

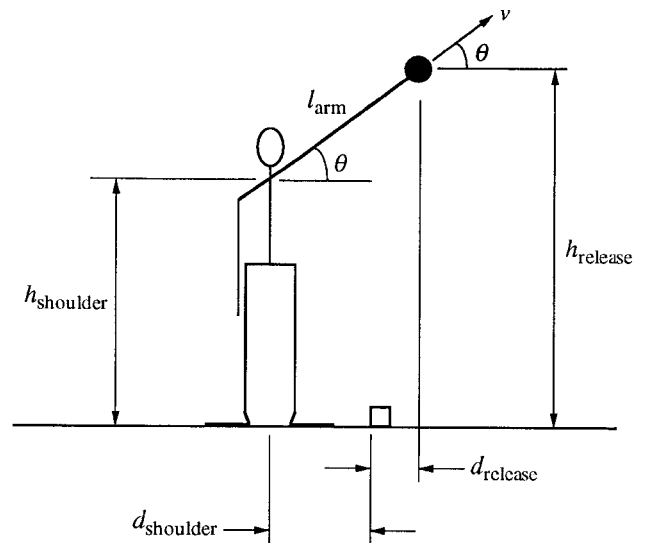


Fig. 5. Anthropometric model of a shot-putter at the instant of release.

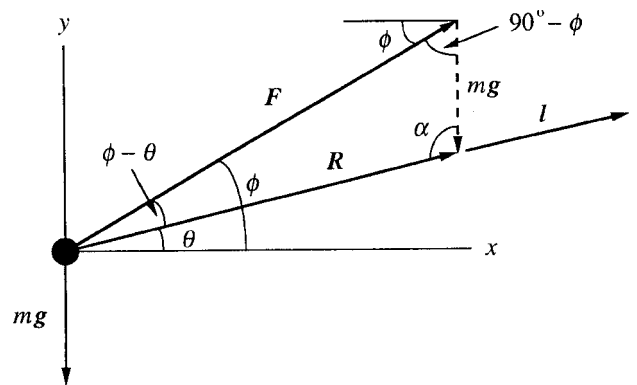


Fig. 6. Model of forces acting on the shot during the delivery phase.

required to achieve this distance is about $13 \text{ m} \cdot \text{s}^{-1}$. When throwing with a preliminary glide or rotational movement, the horizontal velocity of the shot at the end of the preliminary movement is about $2.5 \text{ m} \cdot \text{s}^{-1}$. If the speed of the preliminary movement could be summed without losses to the speed produced in the delivery phase, the release speed would be about $15.5 \text{ m} \cdot \text{s}^{-1}$ and the athlete would achieve a distance of 26 m. However, such summing does not occur. Speed is partly lost and the athlete has a release speed of only about $14 \text{ m} \cdot \text{s}^{-1}$. The distance achieved by the athlete is then about 22 m. In modelling the generation of release speed, it is therefore not essential to include the preliminary phase because this phase has a minor influence on the total distance achieved.

In throws by world-class shot-putters, the trajectory of the shot in the delivery phase is slightly curvilinear, particularly at the start of the movement (Bartonietz

and Borgström, 1995). For simplicity, the delivery path in the model is assumed to be a straight line. Again for simplicity, the force exerted by the athlete on the shot is assumed to be constant throughout the delivery phase.

The model of the delivery phase must be able to account for the decrease in release speed with increasing release angle as observed by Maheras (1995). Zatsiorsky and Matveev (1969) and Hay (1973) suggested that this relation arises because the athlete's muscular forces must overcome the weight of the shot, as well as the inertia of the shot (the resistance of the shot to being accelerated). As the release angle is increased, a greater fraction of the athlete's muscular force is required to overcome the weight of the shot, and so less force is spent accelerating the shot. That is, more of the work performed by the athlete's muscles is required to increase the gravitational potential energy of the shot, at the expense of increasing the kinetic energy of the shot. The release speed that the athlete is able to produce at a high release angle is, therefore, not as great as that at a low release angle.

Applying the principle of conservation of energy to the delivery phase gives

$$W = \Delta KE + \Delta PE \quad (5)$$

where W is the work performed by the athlete's muscles on the shot, ΔKE is the change in kinetic energy of the shot and ΔPE is the change in gravitational potential energy of the shot. In this model, the change in the vertical height of the shot during the delivery phase is $l \sin \theta$, and so the change in the potential energy of the shot is $\Delta PE = mgl \sin \theta$, where m is the mass of the shot. The kinetic energies of the shot at the start and end of the delivery phase are $\frac{1}{2}mv_i^2$ and $\frac{1}{2}mv^2$ respectively, where v_i is the speed of the shot at the start of the delivery phase. The conservation of energy relation for the delivery phase becomes:

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 + mgl \sin \theta \quad (6)$$

Rearranging equation (6) gives an expression for the release speed as a function of release angle:

$$v(\theta) = \left(\frac{2W}{m} - 2gl \sin \theta + v_i^2 \right)^{1/2} \quad (7)$$

Two models of the force exerted by the athlete on the shot were considered. In Model A, the force is constant and independent of the release angle; in Model B, the force decreases linearly with increasing release angle.

Model A: Force is independent of release angle

Here, the force exerted by the athlete on the shot is assumed to be constant and the same for all release

angles. The work done by the athlete on the shot during the delivery phase is (see Appendix):

$$W = Fl \left[1 - \left(\frac{mg}{F} \right)^{1/2} \sin^2 (90^\circ + \theta) \right]^{1/2} \quad (8)$$

Substituting equation (8) into equation (7) gives:

$$v(\theta) = \left\{ \frac{2Fl}{m} \left[1 - \left(\frac{mg}{F} \right)^{1/2} \sin^2 (90^\circ + \theta) \right]^{1/2} - 2gl \sin \theta + v_i^2 \right\}^{1/2} \quad (9)$$

For a world-class male shot-putter, the speed of the shot at the start of the delivery phase (v_i) is about $2.5 \text{ m} \cdot \text{s}^{-1}$ and the length of the acceleration path of the shot (l) is about 1.65 m (Susanka and Stepanek, 1988). Bartonietz (1990) reported a throw by a world-class male shot-putter in which the average force (F) exerted by the athlete on the shot during the delivery phase was about 400 N.

Model B: Force decreases at higher release angles

The assumption that the force exerted by the athlete on the shot is the same at all release angles is unlikely to be realistic. McCoy *et al.* (1984) and Gregor *et al.* (1990) suggested that greater release speeds are produced at low release angles because shot-putters are able to produce more force when projecting the shot at low release angles. They proposed that throwing at a low release angle more closely simulates the arm and trunk position during a bench press movement, and that shot-putters have been conditioned for this exercise more than for the shoulder press exercise, which would simulate throwing at a high release angle.

McWatt (1982) reported values for untrained individuals showing that isometric shoulder strength in a pushing action is greatest when an individual exerts force at an angle of about $10\text{--}20^\circ$ above the line perpendicular to the trunk. At first glance, this implies that shot-putters will have the greatest strength for a release angle of about $10\text{--}20^\circ$. However, McWatt (1982) noted that, when projecting the shot at their competition release angles (around 37°), many athletes bend the trunk back slightly, thus placing their arm in the strongest position to push the shot away from the shoulder. In addition, the delivery phase is more than just a push from the shoulder as it also involves movement of the legs, hips and trunk.

Most athletes probably exert the greatest force when the shot is thrown close to horizontal and exert considerably less force for throws in the vertical direction. In the model used here, the force exerted by the athlete

on the shot was assumed to decrease linearly with release angle:

$$F = F_0 - a\theta \tag{10}$$

where F_0 is the force exerted on the shot for a horizontal release angle and a is a constant that characterizes the athlete's decrease in force with increasing release angle. Substituting equation (10) into equation (9) gives:

$$v(\theta) = \left\{ \frac{2(F_0 - a\theta)l}{m} \left[1 - \left(\frac{mg}{F_0 - a\theta} \right)^2 \sin^2 (90^\circ + \theta) \right] - 2gl \sin \theta + v_i^2 \right\}^{1/2} \tag{11}$$

The constant a is expected to be specific to the individual. It should depend on the athlete's body dimensions, muscle strength and throwing technique. For a world-class shot-putter, the difference in weight lifted between a (two-handed) bench press exercise (200 kg at $\theta \approx 0^\circ$) and a shoulder press exercise (150 kg at $\theta \approx 90^\circ$) suggests a value of about $a = 5 \text{ N} \cdot \text{degree}^{-1}$. However, this suggested value should depend on the familiarity of the athlete with these two strength training exercises.

Results

Maheras' (1995) measurements of the release phase

Maheras (1995) measured the relations between release speed, height and angle for five male college shot-putters. All athletes used the rotational technique and threw from a regular shot-put circle. Each athlete threw 10 times at each of five angles of release for a total of 50 throws. The athletes were instructed to perform maximum-effort throws at their 'normal' release angles and at angles that were 'much lower', 'slightly lower', 'slightly higher' and 'much higher' than their normal release angles. The throws were recorded with a two-dimensional filming procedure using a Pulnix

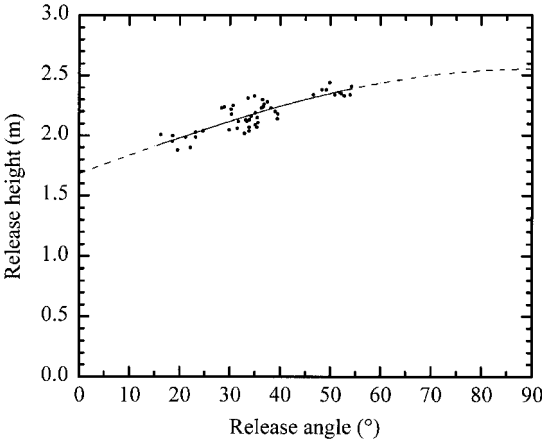


Fig. 7. Release height as a function of release angle for a male college shot-putter (Athlete 1). Data from Maheras (1995).

high-speed video camera operating at 120 Hz and the video recordings were analysed with a Peak Performance biomechanical analysis system.

Table 1 lists the average official distance, release speed, release angle and release height for the 10 throws at the athlete's preferred (i.e. normal) release angle. Figures 7 and 8 show release height and speed as a function of the release angle for the best-performing athlete (Athlete 1). Plots similar to those in Figs 7 and 8 were obtained for the other four athletes in Maheras' study.

Evaluation of the model of the release height

For all five athletes in Maheras' study, a curve of the form of equation (3) was fitted to a plot of the athlete's release height as a function of release angle. The curve was fitted by selecting values of h_{shoulder} and l_{arm} using the Levenberg-Marquardt algorithm (Press *et al.*, 1988). Table 2 lists the calculated values of h_{shoulder} and l_{arm} and Fig. 7 shows the fitted curve for Athlete 1.

The values of h_{shoulder} and l_{arm} were compared to those obtained from anthropometric measurements of the athletes. Maheras did not report limb segment

Table 1. Release parameters for throws at each athlete's preferred release angle (mean \pm s)

Athlete	Official distance, d_{official} (m)	Release speed, v ($\text{m} \cdot \text{s}^{-1}$)	Release angle, θ ($^\circ$)	Release height, h_{release} (m)
1	15.9 \pm 0.8	11.9 \pm 0.3	34.1 \pm 1.5	2.11 \pm 0.05
2	15.6 \pm 0.6	11.8 \pm 0.3	31.0 \pm 1.9	2.06 \pm 0.05
3	15.1 \pm 0.8	11.5 \pm 0.5	34.9 \pm 2.6	2.16 \pm 0.06
4	14.6 \pm 0.6	11.2 \pm 0.4	36.4 \pm 2.4	2.13 \pm 0.08
5	13.8 \pm 0.3	11.1 \pm 0.4	31.6 \pm 1.7	2.09 \pm 0.05

Source: Maheras (1995).

Table 2. Anthropometric parameter values calculated by fitting equation (3) to plots of release height as a function of release angle (values calculated from the height of the athlete are shown for comparison; value \pm standard error)

Athlete	Calculated from equation (3)			Calculated from height of athlete	
	Height* (m)	h_{shoulder} (m)	l_{arm} (m)	h_{shoulder} (m)	l_{arm} (h)
1	1.80	1.68 ± 0.05	0.87 ± 0.08	1.47 ± 0.02	1.02 ± 0.02
2	1.84	1.55 ± 0.03	1.04 ± 0.06	1.50 ± 0.02	1.05 ± 0.02
3	1.83	1.60 ± 0.04	0.98 ± 0.07	1.50 ± 0.02	1.04 ± 0.02
4	1.82	1.60 ± 0.09	0.93 ± 0.15	1.49 ± 0.02	1.04 ± 0.02
5	1.77	1.50 ± 0.04	1.13 ± 0.07	1.45 ± 0.02	1.01 ± 0.02

* Source: A.V. Maheras (personal communication).

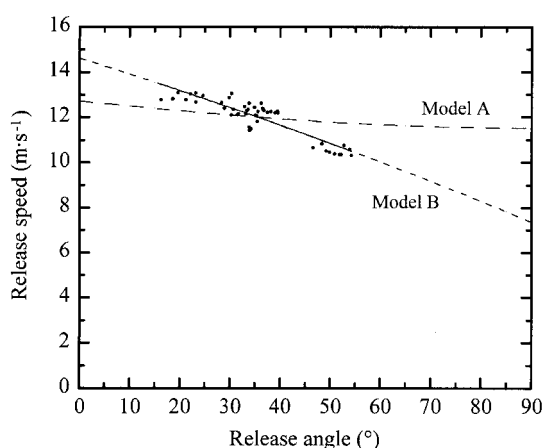


Fig. 8. Release speed as a function of release angle for a male college shot-putter (Athlete 1). Data from Maheras (1995). The fitted curves are for Model A, where the force exerted by the athlete is independent of the release angle, and for Model B, where the force exerted by the athlete decreases linearly with release angle.

lengths for the athletes in his study, but the height of the midpoint of the shoulders (h_{shoulder}) and the length of the throwing arm and shoulder (l_{arm}) for the athletes was estimated from their body heights. According to data presented in Winter (1990), the height of the shoulders when standing upright is 81.8% of body height, and the distance from the midpoint of the shoulders to the tip of the outstretched arm is 56.9% of body height.

The values of h_{shoulder} obtained from the fitted curves are 5–21 cm greater than the height of the shoulders estimated from the body height of the athlete (Table 2). This may be because the athletes were airborne or standing on tip-toe at the instant of release, rather than flat-footed as assumed in the model. For Athletes 2, 3 and 4, the values of l_{arm} obtained from the fitted curves are in good agreement with the length of the throwing arm and shoulder estimated from the body height of the

athlete. The slightly anomalous values for Athletes 1 and 5 suggest that they did not maintain the same body position at all release angles.

Evaluation of the models of the release speed

Model A, in which force is independent of release angle, was *not* an accurate description of the generation of force by the athlete during the delivery phase. For each athlete, a curve of the form of equation (9) was fitted to a plot of the athlete's release speed as a function of release angle. Even with three parameters to select (F , l and v_i), the fitted curves were a poor match to the experimental data. Model A was able to account for only about 20–40% of the observed decrease in release speed with increasing release angle (see Fig. 8). That is, there must be a mechanism, in addition to the weight of the shot, that reduces the speed that the athlete is able to generate as the release angle is increased.

Model B, in which the force exerted by the athlete decreased linearly with release angle, was a much more accurate description of the generation of force by the athlete during the delivery phase. For each athlete, a curve of the form of equation (11) was fitted to a plot of the athlete's release speed as a function of release angle. However, with four fitted parameters (F_0 , a , l and v_i), the uncertainty in the calculated value was several thousand times the value of the parameter.

Better results were obtained with just two fitted parameters (F_0 and a). Maheras (1995) did not measure the path length of the delivery phase or the speed of the shot at the start of the delivery phase. These parameters were set at $l = 1.65$ m and $v_i = 2.5$ m·s⁻¹ to be representative of a world-class male shot-putter (Susanka and Stepanek, 1988) and curves were again fitted to the plots of release speed as a function of release angle. Table 3 lists the calculated values of F_0 and a and Fig. 8 shows the fitted curve for Athlete 1.

The calculated values of a indicate that the force

Table 3. Force parameter values obtained from fitting equation (11) to plots of the release speed as a function of release angle (value \pm standard error)

Athlete	Force at $\theta = 0^\circ$, F_0 (N)	Rate of force decrease, a (N \cdot degree $^{-1}$)
1	462 \pm 11	3.2 \pm 0.3
2	365 \pm 14	1.6 \pm 0.4
3	392 \pm 15	2.4 \pm 0.4
4	338 \pm 20	1.6 \pm 0.5
5	407 \pm 13	3.7 \pm 0.4

exerted by the athlete on the shot is 40–80% less for a vertical angle than for a horizontal release angle. The calculated values of F_0 were close to the expected values for a male college shot-putter. Bartonietz (1990) reported a throw of 21.56 m by a world-class male shot-putter in which the average force exerted by the athlete on the shot during the delivery phase was about 400 N. The athletes in the present study produced throws of 13.8–15.9 m, so the force exerted by the athletes when throwing at their preferred release angle was expected to be about 260–300 N. Taking into account the decrease in force with increasing release angle, the calculated values of F_0 for the athletes in this study were expected to be about 330–400 N.

The force exerted by the athlete on the shot during the delivery phase is much less than the force the athlete can exert in an isometric contraction. Zatsiorsky (1995) noted that a world-class male shot-putter can exert an isometric force of about 1100–1200 N with his throwing arm. The difference between static and dynamic force production is a well-known phenomenon owing to the force–velocity relation of contracting muscle, and has been labelled the ‘explosive strength deficit’.

The calculated values of F_0 and a were insensitive to the choice of the speed of the shot at the start of the delivery phase (v_i). Setting the speed to $v_i = 0 \text{ m} \cdot \text{s}^{-1}$ increased the calculated values of F_0 by less than 4% and decreased the calculated values of a by less than 0.3%. Therefore, the v_i term in the model of the delivery phase (see equations 6, 7, 9 and 11) could be removed without seriously affecting the calculated values of F_0 and a . This confirms that it is not essential to include the preliminary phase in a model of shot-putting.

The optimum release angle

The optimum release angle for each of the athletes in Maheras’ (1995) study was calculated using the methods of Lichtenberg and Wills (1978) and of Red

Table 4. Comparison of the athletes’ preferred release angle with the optimum release angle calculated using the method of Red and Zogaib (1977) and the method of Lichtenberg and Wills (1978) (value \pm standard error)

Athlete	Preferred release angle, θ ($^\circ$)	Optimum release angle, Red and Zogaib ($^\circ$)	Optimum release angle, Lichtenberg and Wills ($^\circ$)
1	34.1 \pm 1.5	31.3 \pm 1.2	41.3 \pm 1.1
2	31.0 \pm 1.9	33.8 \pm 2.0	41.4 \pm 1.1
3	34.9 \pm 2.6	31.4 \pm 2.1	41.1 \pm 1.7
4	36.4 \pm 2.4	32.5 \pm 3.0	40.9 \pm 1.6
5	31.6 \pm 1.7	27.6 \pm 1.8	40.9 \pm 1.5

and Zogaib (1977). In the method of Lichtenberg and Wills, the average values of release speed and height for the 10 ‘normal’ throws were substituted into equations (1) and (2). In the method of Red and Zogaib, the values of h_{shoulder} and l_{arm} were substituted into equation (4) and the values of F_0 and a were substituted into equation (11). The resulting expressions for $h(\theta)$ and $v(\theta)$ were then substituted into equation (1). In all calculations, the radius of the shot was 6.0 cm.

The optimum release angles calculated using the two methods are listed in Table 4. The standard error associated with the calculated optimum release angle was determined using the quadrature method of combining errors (Taylor, 1997). The results confirmed that the method of Red and Zogaib gives an accurate prediction of a shot-putter’s optimum release angle. The athletes’ preferred release angles were in good agreement with the optimum release angles calculated using the method of Red and Zogaib, but were substantially lower than those calculated using the method of Lichtenberg and Wills.

In the method of Red and Zogaib, the change in the athlete’s release speed with release angle, $v(\theta)$, has a much greater effect on the optimum release angle than the change in release height, $h(\theta)$. Figure 9 illustrates how the relations between release speed, height and angle determine the optimum release angle. As the release angle is raised, the release speed that the athlete is able to generate decreases and so the flight distance achieved (solid line) tends to become less than if the release speed were held constant (dashed lines). On the other hand, the increase in the athlete’s release height with increasing release angle tends to increase the flight distance. However, the decrease in release speed has a much greater effect than the increase in release height, and so the overall optimum release angle (31°) is lower than for a projectile released at constant speed and height (41°).

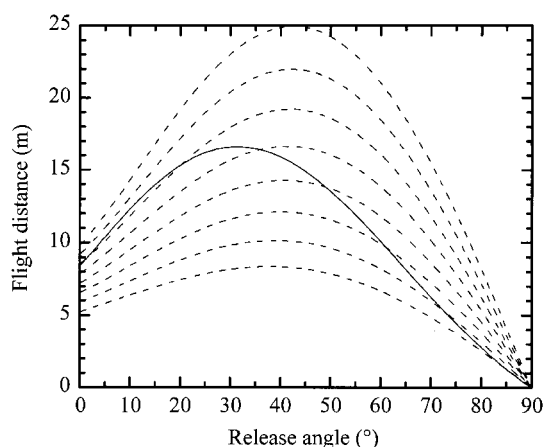


Fig. 9. Calculated flight distance as a function of release angle for a male college shot-putter (Athlete 1). The calculations are based on the method of Red and Zogaib (1977), where the relations between release speed, height and angle for the athlete are combined with the equation of the range of a projectile in free flight.

An alternative way of explaining the optimum release angle is as follows. For a projectile that is released with constant speed from ground level, the optimum release angle that maximizes the flight distance is 45° . However, in shot-putting, there are four additional effects that substantially modify this optimum release angle:

1. The shot is released from above ground level, usually from a height of about 2 m. For the male college shot-putters in this study, the height difference between the release and the landing reduces the optimum release angle by 4° to about 41° .
2. Release height is not exactly constant for all release angles. However, the slight increase in release height with increasing release angle produces a relatively minor effect, raising the optimum release angle by only about 0.5° .
3. As the release angle is increased, more of the athlete's muscular force is required to overcome the weight of the shot. The release speed that the athlete is able to produce therefore decreases with increasing release angle. This lowers the optimum release angle by 2° to about 39° .
4. The structure of the human body favours the production of putting force in the horizontal direction more than in the vertical direction. Therefore, the release speed that the athlete is able to produce decreases at high release angles. For the shot-putters in this study, the optimum release angle is lowered by a further $5\text{--}11^\circ$, and so the overall optimum release angles are between 28° and 34° . Each athlete had his own specific optimum release angle that depended on his rate of force decrease with increasing release angle.

Discussion

The models

Shot-putting is a complex three-dimensional movement. In this study, several simplifying assumptions were introduced to render this complex movement amenable to investigation. For example, the force exerted by the athlete on the shot was assumed to be constant throughout the delivery phase and the delivery path was assumed to be a straight line. Also, the force exerted by the athlete on the shot was assumed to decrease linearly with increasing release angle. Despite these simplifications, the models used here produced good fits to the experimental data (as shown in Figs 7 and 8). It must be emphasized that the conclusions of this study are not affected by any shortcomings of the models of shot-putting used to determine $h(\theta)$ and $v(\theta)$. These expressions could also have been obtained by fitting a straight line or a polynomial to the plots of release height and release speed as a function of release angle. For illustration, the calculated optimum release angles obtained by fitting a straight line to the experimental data were within 0.3° of those obtained using the models.

Effects of release distance, landing distance and air resistance

Although the flight distance contributes most to the official distance, a thorough analysis should also consider the effects of the release distance and the landing distance. Unfortunately, Maheras (1995) did not report how the release distance varied with release angle for the athletes in his study. Susanka and Stepanek (1988) reported release distances ranging from $+20$ cm to -4 cm for competition performances by elite male shot-putters, a negative value indicating that the shot was behind the stop board at the instant of release. The release distance is determined by how close the athlete's body is to the stop board during the delivery phase and by the angle of the athlete's throwing arm at the instant of release. Using the model shown in Fig. 5, the relation between release distance and release angle is of the form

$$d_{\text{release}} = d_{\text{shoulder}} + l_{\text{arm}} \cos \theta \quad (12)$$

where d_{shoulder} is the distance from the stop board to the midpoint of the athlete's shoulders at the instant of release. For an athlete 1.80 m tall, the length of the outstretched arm and shoulder is about 1.02 m (Winter, 1990). If d_{shoulder} is -0.70 m, then the release distance will range from about -70 cm for a vertical release angle to about $+30$ cm for a horizontal release angle, with a value of about $+15$ cm at the optimum release angle. Including the release distance in the analysis lowers the calculated optimum release angles by about 0.4° .

The landing distance usually makes a very small contribution to the official distance. It is affected by the firmness of the landing surface, which determines the size of the circular mark left by the fall of the shot. Invariably, the landing distance is negative and has a magnitude less than the radius of the shot. Because the landing distance is relatively very small, any variations with release angle have a negligible effect ($<0.1^\circ$) on the calculated optimum release angle.

Air resistance also has a negligible effect on the calculation of the optimum release angle. For the best-performing athlete in Maheras' (1995) study (Athlete 1), air resistance reduced the flight distance by about 8 cm and lowered the optimum release angle by about 0.1° (Lichtenberg and Wills, 1978). In summary, release distance, landing distance and air resistance all have relatively minor effects on the calculation of the optimum release angle and so do not change the conclusion that the method of Red and Zogaib (1977) gives an accurate prediction of the optimum release angle.

Sensitivity of performance to release angle and release speed

Projecting the shot at the optimum release angle is *not* critical to successful shot-putting. Relatively large errors in release angle are tolerable because, at angles close to the optimum release angle, the flight distance is insensitive to release angle. For illustration, if the release angle is within about 3° of the optimum release angle, then the flight distance will be within 10 cm of the maximum achievable distance (Fig. 9).

It is much more important for an athlete to attain a high release speed than it is to project the shot at the optimum release angle. For all five athletes in the study of Maheras (1995), distance lost owing to inaccuracies in release angle was much less than distance lost owing to variations in release speed. When throwing at the preferred release angle, the deviations from the calculated optimum release angle reduced the flight distance by less than 20 cm, whereas inter-trial variations in release speed produced changes in flight distance of up to 2.0 m. Variations in flight distance of 0.3–2.2 m have been reported for competition performances by world-class shot-putters (Susanka and Stepanek, 1988; Ueya *et al.*, 1991; Bartonietz and Borgström, 1995; Luhtanen *et al.*, 1997). These performance variations are probably due to variations in release speed caused by inconsistencies in throwing technique.

Conclusions

Maheras' (1995) measurements of release speed, height and angle were explained using two simple models of shot-putting. An anthropometric model of a shot-putter

at the instant of release accounted for the observed increase in release height with increasing release angle. The decrease in release speed with increasing release angle was explained using a model of the forces exerted on the shot during the delivery phase. The shot-putter's release speed decreases because the athlete's muscular forces must overcome an increasing effect of the weight of the shot and because the athlete can generate more force in a horizontal direction than in a vertical direction.

For the five male college shot-putters studied, the method of Red and Zogaib (1977) gave an accurate prediction of the optimum release angle. All athletes had their own specific optimum release angle because of individual differences in the rate of decrease in the force they could exert with increasing release angle. To achieve good performances, it is not necessary to throw at very close to the optimum release angle. Throwing with a high release speed is more important to performance than throwing at the optimum release angle.

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Appendix

In the model of the delivery phase shown in Fig. 6, the work done by the athlete in exerting a constant force F to cause acceleration of the shot along a path length l is given by the projection of F onto l :

$$W = F \cdot l$$

$$W = Fl \cos(\varphi - \theta) \quad (13)$$

where $\varphi - \theta$ is the angle included between the directions of F and l .

The trigonometric identity

$$\sin^2 x + \cos^2 x = 1$$

may be rearranged as

$$\cos x = (1 - \sin^2 x)^{1/2}$$

Letting $x = \varphi - \theta$ gives

$$\cos(\varphi - \theta) = [1 - \sin^2(\varphi - \theta)]^{1/2}$$

and so equation (13) may be written as

$$W = Fl[1 - \sin^2(\varphi - \theta)]^{1/2} \quad (14)$$

The law of sines, applied to the triangle formed by the force vectors F , mg and R , gives

$$\frac{\sin(\varphi - \theta)}{mg} = \frac{\sin \alpha}{F}$$

$$\therefore \sin(\varphi - \theta) = \frac{mg}{F} \sin \alpha \quad (15)$$

The sum of the angles internal to the triangle formed by the force vectors F , mg and R must be equal to 180° . That is,

$$\begin{aligned} 180^\circ &= (\varphi - \theta) + (90^\circ - \varphi) + \alpha \\ \therefore \alpha &= 90^\circ + \theta \end{aligned}$$

Equation (15) then becomes

$$\sin(\varphi - \theta) = \frac{mg}{F} \sin(90^\circ + \theta) \quad (16)$$

Substituting equation (16) into (14) gives an expression for the work performed by the athlete in terms of the force, the length of the acceleration path and the release angle:

$$W = Fl \left[1 - \left(\frac{mg}{F} \right)^2 \sin^2(90^\circ + \theta) \right]^{1/2} \quad (17)$$

If the force exerted by the athlete is very much greater than the weight of the shot, then the direction of the acceleration path of the shot is approximately the same as that of the force exerted by the athlete ($\theta \approx \varphi$), and the work done by the athlete on the shot during the delivery phase is $W \approx Fl$.