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Projekt 5.9 Scaling - the Physics of Lilliput

Teksten er et foredrag holdt af Philip Morrison, Massachusetts Institute of Technology i 1968 ved de såkaldte Christmas Lectures. Teksten findes flere steder på nettet, fx : http://www.dinosaurtheory.com/links.html . Projektets tema er skalering. Det er engelsksproget og lægger således op tip et fagligt samarbejde, fx med læsning af uddrag af Gullivers rejser af Swift. Teksten slutter med en række spørgsmål, som kan danne grundlag for en skriftlig eller mundtlig besvarelse. Efter teksten er der en række henvisninger til litteratur ig til nettet, hvor yderligere materialer om skalering kan findes.

The fictional traveler Lemuel Gulliver spent a busy time in a kingdom called Lilliput, where all living things -- men, cattle, trees, grass -- were exactly similar to our world, except that they were all built on the scale of one inch to the foot. Lilliputians were a little under 6 inches high, on the average, and built proportionately just as we are. Gulliver also visited Brobdingnag, the country of the giants, who were exactly like men but 12 times as tall. As Swift described it, daily life in both kingdoms was about like ours (in the 18th century). His commentary on human behavior is still worth reading, but we shall see that people of such sizes just could not have been as he described them.

Long before Swift lived, Galileo understood why very small or very large models of man could not be like us, but apparently Dean Swift had never read what Galileo wrote. One character in Galileo's ``Two New Sciences'' says, ``Now since. . . in geometry, . . . mere size cuts no figure, I do not see that the properties of circles, triangles, cylinders, cones, and other solid figures will change with their size . . . ." But his physicist friend replies, ``The common opinion is here absolutely wrong.'' Let us see why.

We start with the strength of a rope. It is easy to see that if one man who pulls with a certain strength can almost break a certain rope, two such ropes will just withstand the pull of two men. A single large rope with the same total area of cross-section as the two smaller ropes combined will contain just double the number of fibers of one of the small ropes, and it will also do the job. In other words, the breaking strength of a wire or rope is proportional to its area of cross-section, or to the square of its diameter. Experience and theory agree in this conclusion. Furthermore, the same relation holds, not only for ropes or cables supporting a pull, but also for columns or struts supporting a thrust. The thrust which a column will support, comparing only those of a given material, is also proportional to the cross-sectional area of the column.

Now the body of a man or an animal is held up by a set of columns or struts -- the skeleton -- supported by various braces and cables, which are muscles and tendons. But the weight of the body which must be supported is proportional to the amount of flesh and bone present, that is, to the volume.

Let us now compare Gulliver with the Brobdingnagian giant, 12 times his height. Since the giant is exactly like Gulliver in construction, every one of his linear dimensions is 12 times the corresponding one of Gulliver's. Because the **strength** of his columns and braces is proportional to their cross-sectional area and thus to the square of their linear dimension, his bones will be 12^2 or 144 times as strong as Gulliver's. Because his **weight** is proportional to his volume and thus to L^3, it will be 12^3 or 1728 times as great as Gulliver's. So the giant will have a strength-to-weight ratio a dozen times smaller than ours. Just to support his own weight, he would have as much trouble as we should have in carrying 11 men on our back.
In reality, of course, Lilliput and Brobdingnag do not exist. But we can see real effects of a difference in scale if we compare similar animals of very different sizes. The smaller ones are not scale models of the larger ones. Compare the corresponding leg bones of two closely related animals of the deer family: one a tiny gazelle, the other a bison. Notice that the bone of the large animal is not at all similar geometrically to that of the smaller. It is much thicker for its length, thus counteracting the scale change, which would make a strictly similar bone too weak.

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Galileo wrote very clearly on this very point, disproving the possibility of Brobdingnag, or of any normal-looking giants: ". . . if one wishes to maintain in a great giant the same proportion of limb as that found in an ordinary man he must either use a harder and stronger material for making the bones, or he must admit a diminution of strength in comparison with men of medium stature; for if his height be increased inordinately he will fall and be crushed under his own weight. Whereas, if the size of a body be diminished, the strength of that body is not diminished in the same proportion; indeed, the smaller the body the greater its relative strength. Thus a small dog could probably carry on his back two or three dogs of his own size; but I believe that a horse could not carry even one of his own size."

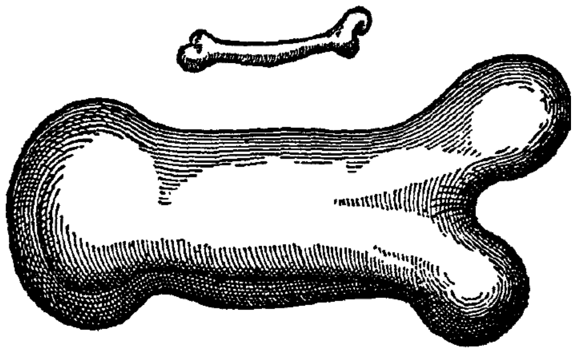

Fig. 27

An elephant is already so large that his limbs are clumsily thickened. However, a whale, the largest of all animals, may weigh 40 times as much as an elephant; yet the whale's bones are not proportionately thickened. They are strong enough because the whale is supported by water. What is the fate of a stranded whale? His ribs break. Some of the dinosaurs of old were animals of whalelike size; how did *they* get along? Following Galileo, we have investigated the problems of scaling up to giants. Now let's take a look at some of the problems that arise when we scale down.

When you climb dripping wet out of a pool there is a thin film of water on your skin. Your fingers are no less wet than your forearm; the thickness of the water film is much the same over most of your body. Roughly, at least, the amount of water you bring out is proportional to the surface area of your body.

Amount of water

L is your height. The original load on your frame is as before, proportional to your volume. So, the ratio *extra load/original load* is proportional to L^2/L^3, or to $1/L$. Perhaps you carry out of the pool a glassful or so, which amounts to about a 1% increase in what you have to move about. But a Lilliputian will bring out about 12% of his weight, which would be equivalent to a heavy winter suit of clothing with an overcoat. Getting out of the pool would be no fun! If a fly gets wet, his body load doubles, and he is all but imprisoned by the drop of water.

Heat loss

There is a still more important effect of the scale of a living body. Your body loses heat mainly through the skin (and some through breathing out warm air). It is easy to believe -- and it can be checked by experiment -- that the heat loss is proportional to the surface area, keeping other factors, like the temperature, nature of skin, and so on, constant. The food taken in must supply this heat, as well as the surplus energy we use in moving about. So minimum food needs go as L^2. If a man like Gulliver can live off a leg of lamb and a loaf of bread for a day or two, a Lilliputian with the same body temperature will require a volume of food only as large. But his leg of lamb, scaled down to his world, will be smaller in volume by a factor of. Therefore, he would need a dozen of his roasts and loaves to feel as well fed as Gulliver did after one. Lilliputians must be a hungry lot, restless, active, graceful, but easily waterlogged. You can recognize these qualities in many small mammals, like a mouse.

We can see why there are no warm-blooded animals much smaller than the mouse. Fish and frogs and insects can be very much smaller because their temperature is not higher than their surroundings. In accord with the scaling laws of area and volume, small, warm-blooded animals need relatively a great deal of food; really small ones could not gather or even digest such an enormous amount. Certainly the agriculture of the Lilliputians could not have supported a kingdom like the one Gulliver described.
Now we see that neither Brobdingnag nor Lilliput can really be a scale model of our world. But what have these conclusions to do with physics?

Let's start again with the very large. As we scale up any system, the load will eventually be greater than the strength of the structure. This effect applies to every physical system, not just to animals, of course. Buildings can be very large because their materials are stronger than bone, their shapes are different, and they do not move. These facts determine the constants like K in the equation

strength = KL^2

but the same laws hold. No building can be made which will look like the Empire State but be as high as a mountain, say 10 000 m. Mountains are solid structures, for the most part, without interior cavities. Just as the bones of a giant must be thick, an object of mountainous size on the earth must be all but solid, or else built of new materials yet unknown.

Our arguments are not restricted to the surface of the Earth. We can imagine building a tremendous structure far out in space away from the gravitational pull of the earth. The load then is not given by the Earth's gravitational pull, but as the structure is built larger and larger each part pulls gravitationally on every other and soon the outside of the structure is pulled in with great force. The inside, built of ordinary materials, is crushed, and large protuberances on the surface break off or sink in. As a result any large structure like a planet has a simple shape, and if it is large enough, the shape is close to a sphere. Any other shape will be unable to support itself. Here is the essential reason why the planets and the Sun tend to be spherical. The pull of gravity is important for us on earth, but as we extend the range of dimensions which we study, it becomes absolutely dominant in the very large. Only motion can change this result. The great masses of gas which are nebulae, for example, are changing in time, and hence the law that large objects must be simple in shape is modified.

When we go from our size to the very small, gravitational effects cease to be important. But as we saw in investigating Lilliput, surface effects become significant. If we go far enough toward the very small, surfaces no longer appear smooth, but are so rough that we have difficulty in defining a surface. Other descriptions must be used. In any case, it will not come as a complete surprise that in the domain of the atom, the very small, scale factors demonstrate that the dominant pull is one which is not easily observed in everyday experience.

Such arguments as these run through all of physics. Like order-of-magnitude measurements, they are extremely valuable when we begin the study of any physical system. How the behavior of a system will change with changes in the scale of its dimensions, its motion, and so on, is often the best guide to a detailed analysis.

Even more, it is by the study of systems built on many unusual scales that physicists have been able to uncover unsuspected physical relations. When changing scale, one aspect of the physical world may be much emphasized and another one may be minimized. In this way we may discover, or at least get a clearer view of, things which are less obvious on our normal scale of experience. It is largely for this reason that physicists examine, in and out of their laboratories, the very large and the very small, the slow and the rapid, the hot and the cold, and all the other unusual circumstances they can contrive. In examining what happens in these circumstances we use instruments both to produce the unusual circumstances and to extend our senses in making measurements.

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It is hard to resist pointing out how much the scale of man's own size affects the way he sees the world. It has been largely the task of physics to try to form a picture of the world which does not depend upon the way we happen to be built. But it is hard to get rid of these effects of our own scale. We can build big roads and bridges which are long and thin, but are essentially not three-dimensional, complex structures. The very biggest things we can make which have some roundness, which are fully three-dimensional, are buildings and great ships. These lack a good deal of being a thousand times larger than men in their linear dimensions.

Within our present technology our scaling arguments are important. If we design a new large object on the basis of a small one, we are warned that new effects too small to detect on our scale may enter and even become the most important things to consider. We cannot just scale up and down blindly, geometrically, but by scaling in the light of physical reasoning, we can sometimes foresee what changes will occur. In this way we can employ scaling in intelligent airplane design, for example, and not arrive at a jet transport that looks like a bee -- and won't fly.

Essay Questions

1. The leg bones of one animal are twice as strong as those of another closely related animal of similar shape. (a) What would you expect to be the ratio of these animals' heights? (b) What would you expect to be the ratio of their weights?
2. A hummingbird must eat very frequently and even then must have a highly concentrated form of food such as sugar. What does the concept of scaling tell you about the size of a hummingbird?
3. About how many Lilliputians would it take to equal the mass of one citizen of Brobdingnag?

Essay Problems

1. The total surface area of a rectangular solid is the sum of the areas of the six faces. If each dimension of a given rectangular solid is doubled, what effect does this have on the total surface area?
2. A hollow metal sphere has a wall thickness of 2 cm. If you increase both the diameter and thickness of this sphere so that the overall volume is three times the original overall volume, how thick will the shell of the new sphere be?
3. If your height and all your other dimensions were doubled, by what factor would this change (a) your weight? (b) the ability of your leg bones to support your weight?
4. According to the zoo, an elephant of mass 4.0×10^3 kg consumes 3.4×10^2 times as much food as a guinea pig of mass 0.70 kg. They are both warm-blooded, plant-eating, similarly shaped animals. Find the ratio of their surface areas, which is approximately the ratio of their heat losses, and compare it with the known ratio of food consumed.
5. A rectangular water tank is supported above the ground by four pillars 5.0 m long whose diameters are 20 cm. If the tank were made 10 times longer, wider, and deeper, what diameter pillars would be needed? How much more water would the tank hold?
6. How many state maps of scale 1:1 000 000 would you need to cover the state with those maps?

*Adapted from **PSSC PHYSICS**, 2nd edition, 1965; D.C. Heath and Company with Education Development Center, Inc., Newton, MA.

Suggested Readings

Galilei, Galileo, "Dialogues Concerning Two New Sciences,'' trans. by Henry Crew and Alphonso De Salvio, Evanston, Northwestern University Press, 1946, pp. 1 - 6, 125 - 128.
Haldane, J. B. S., "On Being the Right Size.'' *World of Mathematics*, Vol. II, edited by James R. Newman. New York, Simon & Schuster, 1956.
Holcomb, Donald F. and Philip Morrison, *My Father's Watch -- Aspects of the Physical World*, Englewood Cliffs, NJ, Prentice Hall, 1974, pp. 68 - 83.
Smith, Cyril S., "The Shape of Things,'' *Scientific American*, January, 1954, p. 58.
Thompson, D'Arcy W., "On Magnitude,'' in *On Growth and Form*, Cambridge University Press, 1952 and 1961.

En fremragende og meget pædagogisk introduktion til skalering, med et væld af eksempler:
http://www.av8n.com/physics/scaling.htm

En side med et væld af eksempler, der kan anvendes i små forløb. Projektets tekst er hentet fra denne side:
http://www.dinosaurtheory.com/links.html

En artikel om flyveøglerne på dinosaurernes tid, der stiller spørgsmålet: hvordan kunne quetzalcoatlus flyve? Det viser sig at være et meget vanskeligt spørgsmål, som kunne danne baggrund for et srudieretningsprojekt. Artiklen kan hentes **her.**